Dynamical Gauge Coupling Constants*

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Abstract

In string theory the coupling parameters are functions of moduli fields. The actual values of the coupling constants are then dynamically determined through the vacuum expextation values of these fields. We review the attempts to connect such theories to low energy effective field theories with realistic gauge coupling constants. This includes a discussion of supersymmetry break-down, the question of a running dilaton, string threshold calculations and the possibility of string unification.

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1 Introduction

Most physics models contain coupling constants as free parameters that can be adjusted to fit the experimental values. In more complete theories one could imagine that the values of these parameters are determined dynamically by the theory itself. The question arises, how and why the coupling parameters take the values that are observed in nature.

String theory[1] provides an example for such a class of models. Couplings are functions of so-called moduli fields and the actual values of the coupling parameters are determined through the vacuum expectation values (vev) of these fields. At tree level the universal gauge coupling constant g_{string} is determined by the vev of the dilaton field[2]

$$S = \frac{4\pi}{g_{\text{string}}^2} + i\frac{\theta}{2\pi}. \tag{1}$$

Nonuniversalities can appear at the one-loop level and depend on further moduli fields T, U or B

$$\frac{1}{g_a^2(\mu)} = \frac{k_a}{g_{\text{string}}^2} + \frac{b_a}{16\pi^2}\ln\frac{M_{\text{string}}^2}{\mu^2} - \frac{1}{16\pi^2}\triangle(T, U, B \ \ldots) \ . \tag{2}$$

Given this situation we have then to see how a theory with a realistic set of gauge coupling constants can emerge.

We would therefore like to connect such a theory with a low energy effective field theory describing the known particle physics phenomena. A prime candidate is a low energy supergravitational generalization of the standard model of strong and electroweak interactions, with supersymmetry broken in a hidden sector[3]. Unification of observable sector gauge couplings might appear as proposed in supersymmetric grand unified theories (SUSYGUTs)[4]. The vevs of the moduli fields S, T, ... should then determine gauge couplings in hidden and observable sector, including the correct values of the QCD coupling α_s and the weak mixing angle $\sin^2\theta_{\text{W}}$.

In string theories with unbroken supersymmetry the vevs of the moduli fields are undetermined. A first step in the determination of gauge coupling constants requires therefore the discussion of supersymmetry breakdown. We then have to see how the vevs of the moduli are fixed. Of course, not any value of the moduli will lead to a satisfactory model. In fact we shall face some generic problems concerning the actual values of the coupling constants. We have to understand why the value of

$$\alpha_{\text{string}} = \frac{g_{\text{string}}^2}{4\pi} \approx \frac{1}{20}, \tag{3}$$

whereas a natural expectation for the vev of S would be a number of order 1 or maybe 0 or even infinity, as happens in many simple models. A related question concerns the possibility of so-called string unification leading to the correct prediction of the

weak mixing angle $\sin^2 \theta_W$. Naively one would expect string unification to appear at $M_{\text{string}} \approx 4 \times 10^{17} \text{GeV}$, while the correct prediction of $\sin^2 \theta_W$ seems to lead to a scale that is a factor of 20 smaller. We then have to face the question how such a situation can be achieved with natural values of the vevs of the moduli fields.

These are the questions we want to address in these lectures. We shall start with the discussion of supersymmetry breakdown in the framework of gaugino condensation. This includes a discussion of the problem of a "runaway" dilaton that any attempt of a dynamical determination of coupling constants has to face. The next question then concerns the value of $< S > \approx 1$ and its compatibility with weak coupling. Finally we shall discuss new results concerning string threshold corrections and the question of string unification[5].

2 Gaugino Condensation

One of the prime motivations to consider the supersymmetric extension of the standard model is the stability of the weak scale (M_W) of order of a TeV in the presence of larger mass scales like a GUT-scale of $M_X \approx 10^{16} \ GeV$ or the Planck scale $M_{Pl} \approx 10^{18} \ GeV$. The size of the weak scale is directly related to the breakdown scale of supersymmetry, and a satisfactory mechanism of supersymmetry breakdown should explain the smallness of M_W/M_{Pl} in a natural way. One such mechanism is based on the dynamical formation of gaugino condensates that has attracted much attention since its original proposal for a spontaneous breakdown of supergravity [6][7]. In the following we shall address some open questions concerning this mechanism in the framework of low energy effective superstring theories. This work has been done in collaboration with Z. Lalak and A. Niemeyer and appeared in ref. [8][9].

Before discussing these detailed questions let us remind you of the basic facts of this mechanism. For simplicity we shall consider here a pure supersymmetric $(N = 1)$ Yang-Mills theory, with the vector multiplet (A_μ, λ) containing gauge bosons and gauge fermions in the adjoint representation of the nonabelian gauge group. Such a theory is asymptotically free and we would therefore (in analogy to QCD) expect confinement and gaugino condensation at low energies [10]. We are then faced with the question whether such a nontrivial gaugino condensate $<\lambda\lambda> \neq 0$ leads to a breakdown of supersymmetry. A first look at the SUSY-transformation on the composite fermion $\lambda\sigma^\mu A_\mu$ [11]

$$\{Q, \lambda\sigma^\mu A_\mu\} = \lambda\lambda + \dots \tag{4}$$

might suggest a positive answer, but a careful inspection of the multiplet structure and gauge invariance leads to the opposite conclusion. The bilinear $\lambda\lambda$ has to be interpreted as the lowest component of the chiral superfield $W^\alpha W_\alpha = (\lambda\lambda, \dots)$ and therefore a non-vanishing vev of $\lambda\lambda$ does not break SUSY [12]. This suggestion is supported by index-arguments [13] and an effective Lagrangian approach [14]. We

are thus lead to the conclusion that in such theories gaugino condensates form, but do not break global (rigid) supersymmetry.

Not all is lost, however, since we are primarily interested in models with local supersymmetry including gravitational interactions. The weak gravitational force should not interfere with the formation of the condensate; we therefore still assume $<\lambda\lambda> = \Lambda^3 \neq 0$. This expectation is confirmed by the explicit consideration of the effective Lagrangian of ref. [6] in the now locally supersymmetric framework. We here consider a composite chiral superfield $U = (u, \psi, F_u)$ with $u = <\lambda\lambda>$. In this toy model [6][7] we obtain the surprising result that not only $<u> = \Lambda^3 \neq 0$ but also $<F_u> \neq 0$, a signal for supersymmetry breakdown. In fact

$$<F_u> = M_S^2 = \frac{\Lambda^3}{M_{Pl}},\tag{5}$$

consistent with our previous result that in the global limit $M_{Pl} \to \infty$ (rigid) supersymmetry is restored. For a hidden sector supergravity model we would choose $M_S \approx 10^{11}\ GeV$ [7].

Still more information can be obtained by consulting the general supergravity Lagrangian of elementary fields determined by the Kähler potential $K(\Phi_i, \Phi^{j*})$, the superpotential $W(\Phi_i)$ and the gauge kinetic function $f(\Phi_i)$ for a set of chiral superfields $\Phi_i = (\phi_i, \psi_i, F_i)$. Non-vanishing vevs of the auxiliary fields F_i would signal a breakdown of supersymmetry. In standard supergravity notation these fields are given by

$$F_i = \exp(G/2)(G^{-1})_i^j G_j + \frac{1}{4}\frac{\partial f}{\partial \Phi_k}(G^{-1})_i^k \lambda\lambda + \dots,\tag{6}$$

where the gaugino bilinear appears in the second term [15]. This confirms our previous argument that $<\lambda\lambda> \neq 0$ leads to a breakdown of supersymmetry, however, we obtain a new condition: $\partial f/\partial\Phi_i$ has to be nonzero, i.e. the gauge kinetic function $f(\Phi_i)$ has to be nontrivial. In the fundamental action $f(\Phi_i)$ multiplies $W_\alpha W^\alpha$ which in components leads to a form $\mathrm{Re}f(\phi_i)F_{\mu\nu}F^{\mu\nu}$ and tells us that the gauge coupling is field dependent. For simplicity we consider here one modulus field M with

$$<\mathrm{Re}f(M)> \approx 1/g^2.\tag{7}$$

This dependence of f on the modulus M is very crucial for SUSY breakdown via gaugino condensation. $\partial f/\partial M \neq 0$ leads to $F_M \approx \Lambda^3/M_{Pl}$ consistent with previous considerations. The goldstino is the fermion in the $f(M)$ supermultiplet. In the full description of the theory it might mix with a composite field, but the inclusion of the composite fields should not alter the qualitative behaviour discussed here. An understanding of the mechanism of SUSY breakdown via gaugino condensation is intimately related to the question of a dynamical determination of the gauge coupling constant as the vev of a modulus field. We would hope that in a more complete theory such questions could be clarified in detail.

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One candidate of such a theory is the $E_8 \times E_8$ heterotic string. The second E_8 (or a subgroup thereof) could serve as the hidden sector gauge group and it was soon found [2] that there we have nontrivial $f = S$ where S represents the dilaton superfield. The heterotic string thus contains all the necessary ingredients for a successful implementation of the mechanism of SUSY breakdown via gaugino condensation [16][17]. Also the question of the dynamical determination of the gauge coupling constant can be addressed. A simple reduction and truncation from the $d = 10$ theory leads to the following scalar potential [18]

$$V = \frac{1}{16 S_R T_R^3} \left[|W(\Phi) + 2(S_R T_R)^{3/2} (\lambda\lambda)|^2 + \frac{T_R}{3} \left| \frac{\partial W}{\partial \Phi} \right|^2 \right], \tag{8}$$

where $S_R = \mathrm{Re} S$, $T_R = \mathrm{Re} T$ is the modulus corresponding to the overall radius of compactification and $W(\Phi)$ is the superpotential depending on the matter fields Φ. The gaugino bilinear appears via the second term in the auxiliary fields (6). To make contact with the dilaton field, observe that $<\lambda\lambda> = \Lambda^3$ where Λ is the renormalization group invariant scale of the nonabelian gauge theory under consideration. In the one-loop approximation

$$\Lambda = \mu \exp\left(-\frac{1}{bg^2(\mu)} \right), \tag{9}$$

with an arbitrary scale μ and the β-function coefficient b. This then suggests

$$\lambda\lambda \approx e^{-f} = e^{-S} \tag{10}$$

as the leading contribution (for weak coupling) for the functional f-dependence of the gaugino bilinear[1].

In the potential (8) we can then insert (10) and determine the minimum. In our simple model (with $\partial W / \partial T = 0$) we have a positive definite potential with vacuum energy $E_{vac} = 0$. Suppose now for the moment that $<W(\Phi)> \neq 0$[2]. S will now adjust its vev in such a way that $|W(\Phi) + 2(S_R T_R)^{3/2} (\lambda\lambda)| = 0$, thus

$$|W(\Phi) + 2(S_R T_R)^{3/2} \exp(-S)| = 0. \tag{11}$$

This then leads to broken SUSY with $E_{vac} = 0$ and a fixed value of the gauge coupling constant $g^2 \approx <\mathrm{Re} S>^{-1}$. For the vevs of the auxiliary fields we obtain $F_S = 0$ and $F_T \neq 0$ with important consequences for the pattern of the soft SUSY breaking terms in phenomenologically oriented models [21], which we shall not discuss here in detail.

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[1] Relation (10) is of course not exact. For different implementations see [17], [19], [20]. The qualitative behaviour of the potential remains unchanged.

[2] In many places in the literature it is quoted incorrectly that $<W(\Phi)>$ is quantized in units of the Planck length since W comes from H, the field strength of the antisymmetric tensor field B and $H = \mathrm{d}B - \omega_{3Y} + \omega_{3L}$ (ω being the Chern-Simons form). Quantization is expected for $<\mathrm{d}B>$ but not necessarily for H.

Thus a satisfactory picture seems to emerge. However, we have just discussed a simplified example. In general we would expect also that the superpotential depends on the moduli, $\partial W/\partial T \neq 0$ and, including this dependence, the modified potential would no longer be positive definite and one would have $E_{vac} < 0$.

But even in the simple case we have a further vacuum degeneracy. For any value of $W(\Phi)$ we obtain a minimum with $E_{vac} = 0$, including $W(\Phi) = 0$. In the latter case this would correspond to $<\lambda\lambda> = 0$ and $S \to \infty$. This is the potential problem of the runaway dilaton. The simple model above does not exclude such a possibility. In fact this problem of the runaway dilaton does not seem just to be a problem of the toy model, but more general. One attempt to avoid this problem was the consideration of several gaugino condensates [22], but it still seems very difficult to produce satisfactory potentials that lead to a dynamical determination of the dilaton for reasonable values of $<S>$. In some cases it even seems impossible to fine tune the cosmological constant to zero. In absence of a completely satisfactory model it is then also difficult to investigate the detailed phenomenological properties of the approach. Here it would be of interest to know the actual size of the vevs of the auxiliary fields $<F_S>$, $<F_T>$ and $<F_U>$. In the models discussed so far one usually finds $<F_T>$ to be the dominant term, but it still remains a question whether this is true in general.

3   Fixing the dilaton

In any case it seems that we need some new ingredient before we can understand the mechanism completely. It is our belief, that the resolution of all these problems comes with a better understanding of the form of the gauge kinetic function f [8][9]. In all the previous considerations one assumed $f = S$. How general is this relation? Certainly we know that in one loop perturbation theory S mixes with T [23], but this is not relevant for our discussion and, for simplicity, we shall ignore that for the moment. The formal relation between f and the condensate is given through $\Lambda^3 \approx e^{-f}$ and we have $f = S$ in the weak coupling limit of string theory. In fact this argument tells us only that

$$\lim_{S\to\infty} f(S) = S. \tag{12}$$

Nonperturbative effects could lead to the situation that f is a very complicated function of S. In fact a satisfactory incorporation of gaugino condensates in the framework of string theory might very well lead to such a complication. In our work [8] we suggested that a nontrivial f-function is the key ingredient to better understand the mechanism of gaugino condensation. We still assume (12) to make contact with perturbation theory. How do we then control e^{-f} as a function of S? In absence of a determination of $f(S)$ by a direct calculation one might use symmetry arguments to make some progress. Let us here consider the presence of a symmetry called S-duality which in its simplest form is given by a $SL(2, Z)$ generated by the transformations

$$S \to S + i, \quad S \to -1/S. \tag{13}$$

Such a symmetry might be realized in two basically distinct ways: the gauge sector could close under the transformation (type I) or being mapped to an additional 'magnetic sector' with inverted coupling constant (type II). In the second case one would speak of strong-weak coupling duality, just as in the case of electric-magnetic duality [24]. Within the class of theories of type I, however, we could have the situation that the f-function is itself invariant[3] under S-duality; i.e. $S \to -1/S$ does not invert the coupling constant since the gauge coupling constant is not given by ReS but $1/g^2 \approx$ Ref. In view of (12) we would call such a symmetry weak-weak coupling duality. The behaviour of the gauge coupling constant as a function of S is shown in Fig. 1. Our assumption (12) implies that $g^2 \to 0$ as Re$S \to \infty$ and by S-duality g^2 also vanishes for $S \to 0$, with a maximum somewhere in the vicinity of the self-dual point $S = 1$. Observe that $S \approx 1$ in this situation does not necessarily imply strong coupling, because $g^2 \approx 1/\text{Re}f$ and even for $S \approx 1$, Ref could be large and $g^2 << 1$, with perturbation theory valid in the whole range of S. Of course, nonperturbative effects are responsible for the actual form of $f(S)$.
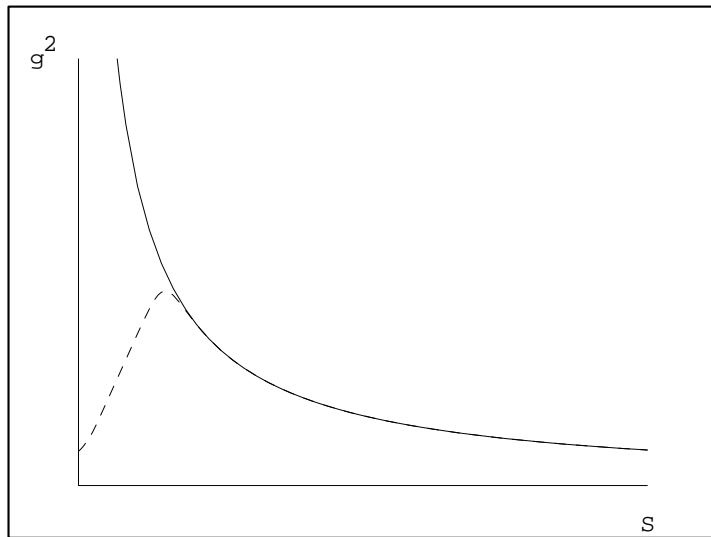

Fig. 1 - Coupling constant g^2 as the function of S in type-I models (dashed) vs g^2 given by $f = S$

To examine the behaviour of the scalar potential in this approach, let us consider a simple toy model, with chiral superfields $U = Y^3 = (\lambda\lambda, \ldots)$ as well as S and T. We have to choose a specific example of a gauge kinetic function which is invariant under the S-duality transformations. Different choices are possible, the simplest is given by

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[3]More complicated choices of transformation properties for f are possible and lead to similar results as obtained in our simple toy model.

$$f = \frac{1}{2\pi} \ln(j(S) - 744), \tag{14}$$

$j(S)$ being the usual generator of modular invariant functions. This function behaves like S in the large S-limit. If we assume a type I-model where the gauge sector is closed under S-duality, then we also have to assume that the gaugino condensate does not transform under S-duality (because of the $fW^\alpha W_\alpha$-term in the Lagrangian)[4]. Under these conditions an obvious candidate for the superpotential is just the standard Veneziano-Yankielowicz superpotential (extended to take into account the usual T-duality, which we assume to be completely independent from S-duality) [14][25]

$$W = Y^3(f + 3b \ln \frac{Y\eta^2(T)}{\mu} + c). \tag{15}$$

This is clearly invariant under S-duality. Therefore we then cannot take the conventional form for the Kähler potential which would be given by

$$K = -\ln(S + \bar{S}) - 3\ln(T + \bar{T} - Y\bar{Y}), \tag{16}$$

since it is not S-dual. To make it S-dual one could introduce an additional $\ln|\eta(S)|^4$ term, giving e.g.

$$K = -\ln(S + \bar{S}) - 3\ln(T + \bar{T} - Y\bar{Y}) - \ln|\eta(S)|^4. \tag{17}$$

Because the only relevant quantity is

$$G = K + \ln|W|^2, \tag{18}$$

we can as well put this new term (which is forced upon us because of our demand for symmetry) into the superpotential and take the canonical Kähler function instead, which gives

$$K = -\ln(S + \bar{S}) - 3\ln(T + \bar{T} - Y\bar{Y}), \tag{19}$$

$$W = \frac{Y^3}{\eta^2(S)}(f + 3b \ln \frac{Y\eta^2(T)}{\mu} + c), \tag{20}$$

where the remarkable similarity to the effective potential for T-dual gaugino condensation $W = W_{inv}/\eta^6(T)$ can be seen more clearly.

This model exhibits a well defined minimum at $<S> = 1$, $<T> = 1.23$ and $<Y> \approx \mu$. Supersymmetry is broken with the dominant contribution being $<F_T> \approx \mu^3$. The cosmological constant is negative.

In contrast to earlier attempts [26] this model fixes the problem of the runaway dilaton and breaks supersymmetry with only a single gaugino condensate. Previous

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[4]For type I-models it was shown in [8] that one can always redefine the gauge kinetic function and condensate in such a way that this holds.

models needed multiple gaugino condensates and (to get realistic vevs for the dilaton) matter fields in complicated representations. We feel that the concept of a nontrivial gauge kinetic function derived (or constrained) by a symmetry is a much more natural way to fix the dilaton and break supersymmetry, especially so because corrections to $f = S$ are expected in any case. Earlier models which included S-duality in different ways (both with and without gaugino condensates) [27][28] were able to fix the vev of the dilaton but did not succeed in breaking supersymmetry. An alternative mechanism to fix the vev of the dilaton has been discussed in [29].

Of course there are still some open questions not solved by this approach. The first is the problem of having a vanishing cosmological constant. Whereas early models of gaugino condensation often introduced *ad hoc* terms to guarantee a vanishing vacuum energy, it has been seen to be notoriously difficult to get this out of models based on string inspired supergravity. The only way out of this problem so far has been to introduce a constant term into the superpotential, parameterizing unknown effects. This approach does not even work in any arbitrary model, but at least in our model the cosmological constant can be made to vanish by adjusting such a constant.

Another question not addressed in this toy model is the mixing of S and T fields which happens at the one-loop level. It is still unknown whether one can keep two independent dualities in this case. In a consistent interpretation our toy model should describe an all-loop effective action. If it is considered to be a theory at the tree-level then the theory is not anomaly free. Introducing terms to cancel the anomaly which arises because of demanding S-duality will then destroy S-duality. At tree-level the theory therefore cannot be made anomaly free.

An additional interesting question concerns the vevs of the auxiliary fields, i.e. which field is responsible for supersymmetry breakdown. In all models considered so far (multiple gaugino condensates, matter, S-duality) it has always been F_T which dominates all the other auxiliary fields. It has not been shown yet that this is indeed a generic feature. The question is an important one, since the hierarchy of the vevs of the auxiliary fields is mirrored in the structure of the soft SUSY breaking terms of the MSSM [21]. We want to argue that there is at least no evidence for F_T being generically large in comparison to F_S, because all of the models constructed so far (including our toy model) are designed in such a way that $<F_S> = 0$ by construction at the minimum (at least at tree-level for the other models). In fact, if one extends our model with a constant in the superpotential (see above), then $<F_S>$ increases with the constant (but does not become as large as $<F_T>$).

Of course there are still some assumptions we made by considering this toy model. We assumed that there is weak coupling in the large S limit which is an assumption because the nonperturbative effects are unknown (at tree-level it can be calculated that $f = S$). In addition it is clear that the standard form we take for the Kähler potential does not include nonperturbative effects and thus could be valid only in the weak coupling approximation (this is of course related to our choice of the superpotential). Of course an equally valid assumption would be that nonperturbative

effects destroy the calculable tree-level behaviour even in the weak coupling region. The model of ref. [27] could be re-interpreted in that sense (they do not consider gaugino condensates and the gauge kinetic function, but their S-dual scalar potential goes to infinity for $S \to \infty$). We choose not to make this assumption, because it is equivalent to the statement that the whole perturbative framework developed so far in string theory is wrong. Again it should be emphasized here that the S-duality considered is not a strong-weak coupling duality but a weak-weak coupling duality. In type II-models one has a duality between strong and weak coupling [8]. A detailed discussion of such a scenario would be desirable.

4   S=1 and weak coupling

A problem could be the actual size of the gauge coupling constant. If $f = S$ and $<S> = 1$ then the large value of the gauge coupling constant does not fit the low scale of gaugino condensation necessary for phenomenologically realistic supersymmetry breaking ($10^{13} \, GeV$). However if $f = S$ only in the weak coupling limit then one can have $<f> >> 1$ and thus $g^2 << 1$ even in the region $S = O(1)$. Therefore in our model $<S> = 1$ is consistent with the demand for a small gauge coupling constant, whereas in models with $f = S$ a much larger (and therefore more unnatural) $<S>$ is needed.

To summarize we find that the choice of a nontrivial f-function (motivated by a symmetry requirement) gives rise to a theory where supersymmetry breaking is achieved by employing only a single gaugino condensate. The cosmological constant turns out to be negative, but can be adjusted by a simple additional constant in the superpotential. The vevs of all fields are at natural orders of magnitude and due to the nontrivial gauge kinetic function the gauge coupling constant can be made small enough to give a realistic picture.

Turning our attention to the observable sector we see that a small (grand unified) coupling constant is a necessity and the above mechanism is required for a satisfactory description of the size af the observed coupling constants like e.g. α_{QCD}. But this alone might not be sufficient for a realistic model. String theory should predict all low energy coupling constants correctly and should also give the correct ratio of electroweak and strong coupling constants.

LEP and SLC high precision electroweak data give for the minimal supersymmetric Standard Model (MSSM) with the lightest Higgs mass in the range 60GeV $< M_{\mathrm{H}} < 150$GeV

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\begin{aligned}
\sin^2 \hat{\theta}_{\mathrm{W}}(M_{\mathrm{Z}}) &= 0.2316 \pm 0.0003 \\
\alpha_{em}(M_{\mathrm{Z}})^{-1} &= 127.9 \pm 0.1 \\
\alpha_{\mathrm{S}}(M_{\mathrm{Z}}) &= 0.12 \pm 0.01 \\
m_t &= 160^{+11+6}_{-12-5} \mathrm{GeV} \ ,
\end{aligned}
\tag{21}
$$

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for the central value $M_{\rm H} = M_{\rm Z}$ in the $\overline{MS}$ scheme [30]. This is in perfect agreement with the recent CDF/D0 measurements of m_t. Taking the first three values as input parameters leads to gauge coupling unification at $M_{\rm GUT} \sim 2 \cdot 10^{16}$GeV with $\alpha_{\rm GUT} \sim \frac{1}{26}$ and $M_{\rm SUSY} \sim 1$TeV [31, 30]. Slight modifications arise from light SUSY thresholds, i.e. the splitting of the sparticle mass spectrum, the variation of the mass of the second Higgs doublet and two–loop effects. Whereas these effects are rather mild, huge corrections may arise from heavy thresholds due to mass splittings at the high scale $M_{heavy} \neq M_{\rm GUT}$ arising from the infinite many massive string states [32]. In the following sections we shall discuss this question of string unification in detail.

5   Gauge coupling unification

In heterotic superstring theories all couplings are related to the universal string coupling constant $g_{\rm string}$ at the string scale $M_{\rm string} \sim 1/\sqrt{\alpha'}$, with α' being the inverse string tension. It is a free parameter which is fixed by the dilaton vacuum expectation value $g_{\rm string}^{-2} = \frac{S + \overline{S}}{2}$. In general this amounts to string unification, i.e. at the string scale $M_{\rm string}$ all gauge and Yukawa couplings are proportional to the string coupling and are therefore related to each other. For the gauge couplings (denoted by g_a) we have [33]:

$$g_a^2 k_a = g_{\rm string}^2 = \frac{\kappa^2}{2\alpha'} \; . \tag{22}$$

Here, k_a is the Kac–Moody level of the group factor labeled by a. The string coupling $g_{\rm string}$ is related to the gravitational coupling constant κ^2. In particular this means that string theory itself provides gauge coupling and Yukawa coupling unification even in absence of a grand unified gauge group.

To make contact with the observable world one constructs the field–theoretical low–energy limit of a string vacuum. This is achieved by integrating out all the massive string modes corresponding to excited string states as well as states with momentum or winding quantum numbers in the internal dimensions. The resulting theory then describes the physics of the massless string excitations at low energies $\mu < M_{\rm string}$ in field–theoretical terms. If one wants to state anything about higher energy scales one has to take into account threshold corrections $\triangle_a(M_{\rm string})$ to the bare couplings $g_a(M_{\rm string})$ due to the infinite tower of massive string modes. They change the relations (22) to:

$$g_a^{-2} = k_a g_{\rm string}^{-2} + \frac{1}{16\pi^2}\triangle_a \; , \tag{23}$$

The corrections in (23) may spoil the string tree–level result (22) and split the one–loop gauge couplings at $M_{\rm string}$. This splitting could allow for an effective unification at a scale $M_{\rm GUT} < M_{\rm string}$ or destroy the unification.

The general expression of $\triangle_a$ for heterotic tachyon–free string vacua is given in [34]. Various contributions to $\triangle_a$ have been determined for several classes of models: First in [34] for two $\mathbb{Z}_3$ orbifold models with a (2,2) world–sheet supersymmetry [35]. This has been extended to fermionic constructions in [36]. Threshold corrections for (0,2) orbifold models with quantized Wilson lines [37] have been calculated in [38]. Threshold corrections for the quintic threefold and other Calabi–Yau manifolds [39] with gauge group $E_6 \times E_8$ can be found in [40, 41]. In toroidal orbifold compactifications [35] moduli dependent threshold corrections arise only from N=2 supersymmetric sectors. They have been determined for some orbifold compactifications in [42]–[45] and for more general orbifolds in [46]. The full moduli dependence[5] of threshold corrections for (0,2) orbifold compactifications with continuous Wilson lines has been first derived in [47, 48]. These models contain continuous background gauge fields in addition to the usual moduli fields [51]. In most of the cases these models are (0,2) compactifications. In all the above orbifold examples the threshold corrections $\triangle_a$ can be decomposed into three parts:

$$\triangle_a = \tilde{\triangle}_a - b_a^{N=2}\triangle + k_a\,Y \ . \tag{24}$$

Here the gauge group dependent part is divided into two pieces: The moduli independent part $\tilde{\triangle}_a$ containing the contribution of the N=1 supersymmetric sectors as well as scheme dependent parts which are proportional to b_a. This prefactor b_a is related to the one–loop β–function: $\beta_a = b_a g_a^3/16\pi^2$. Furthermore the moduli dependent part $b_a^{N=2}\triangle$ with $b_a^{N=2}$ being related to the anomaly coefficient b'_a by $b_a^{N=2} = b'_a - k_a\delta_{\mathrm{GS}}$. The gauge group independent part Y contains the gravitational back–reaction to the background gauge fields as well as other universal parts [34, 52, 41, 53]. They are absorbed into the definition of g_{string}: $g_{\mathrm{string}}^{-2} = \frac{S+\overline{S}}{2} + \frac{1}{16\pi^2}Y$. The scheme dependent parts are the IR–regulators for both field– and string theory as well as the UV–regulator for field theory. The latter is put into the definition of M_{string} in the $\overline{\mathrm{DR}}$ scheme [34]:

$$M_{\mathrm{string}} = 2\frac{e^{(1-\gamma_{\mathrm{E}})/2}3^{-3/4}}{\sqrt{2\pi\alpha'}} = 0.527\,g_{\mathrm{string}} \times 10^{18}\ \mathrm{GeV} \ . \tag{25}$$

The constant of the string IR–regulator as well as the universal part due to gravity were recently determined in [53].

The identities (23) are the key to extract any string–implication for low–energy physics. They serve as boundary conditions for our running field–theoretical couplings valid below M_{string} [54]. Therefore they are the foundation of any discussion about both low–energy predictions and gauge coupling unification. The evolution equations[6] valid below M_{string}

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[5]A lowest expansion result in the Wilson line modulus has been obtained in [49, 50].

[6]We neglect the N=1 part of $\tilde{\triangle}_a$ which is small compared to $b_a^{N=2}\triangle$ [34, 36, 38].

$$\frac{1}{g_a^2(\mu)} = \frac{k_a}{g_{\text{string}}^2} + \frac{b_a}{16\pi^2} \ln \frac{M_{\text{string}}^2}{\mu^2} - \frac{1}{16\pi^2} b_a^{N=2} \triangle \ , \tag{26}$$

allow us to determine $\sin^2 \theta_W$ and α_S at M_Z. After eliminating g_{string} in the second and third equations one obtains

$$
\begin{aligned}
\sin^2 \theta_W(M_Z) &= \frac{k_2}{k_1 + k_2} - \frac{k_1}{k_1 + k_2} \frac{\alpha_{em}(M_Z)}{4\pi} \left[\mathcal{A} \ln \left(\frac{M_{\text{string}}^2}{M_Z^2} \right) - \mathcal{A}' \triangle \right] , \\
\alpha_S^{-1}(M_Z) &= \frac{k_3}{k_1 + k_2} \left[\alpha_{em}^{-1}(M_Z) - \frac{1}{4\pi} \mathcal{B} \ln \left(\frac{M_{\text{string}}^2}{M_Z^2} \right) + \frac{1}{4\pi} \mathcal{B}' \triangle \right] ,
\end{aligned}
\tag{27}
$$

with $\mathcal{A} = \frac{k_2}{k_1} b_1 - b_2, \mathcal{B} = b_1 + b_2 - \frac{k_1+k_2}{k_3} b_3$ and $\mathcal{A}', \mathcal{B}'$ are obtained by exchanging $b_i \to b_i'$. For the MSSM one has $\mathcal{A} = \frac{28}{5}, \mathcal{B} = 20$. However to arrive at the predictions of the MSSM (21) one needs huge string threshold corrections $\triangle$ due to the large value of M_{string} $(3/5k_1 = k_2 = k_3 = 1)$:

$$\triangle = \frac{\mathcal{A}}{\mathcal{A}'} \left[\ln \left(\frac{M_{\text{string}}^2}{M_{\text{GUT}}^2} \right) + \frac{32\pi \delta_{\sin^2 \theta_W}}{5\mathcal{A}\alpha_{em}(M_Z)} \right] . \tag{28}$$

At the same time, the N=2 spectrum of the underlying theory encoded in $\mathcal{A}', \mathcal{B}'$ which enters the threshold corrections has to fulfill the condition

$$\frac{\mathcal{B}'}{\mathcal{A}'} = \frac{\mathcal{B}}{\mathcal{A}} \frac{\ln \left(\frac{M_{\text{string}}^2}{M_{\text{GUT}}^2} \right) + \frac{32\pi}{3\mathcal{B}} \delta_{\alpha_S^{-1}}}{\ln \left(\frac{M_{\text{string}}^2}{M_{\text{GUT}}^2} \right) + \frac{32\pi}{5\mathcal{A}} \frac{\delta_{\sin \theta_W^2}}{\alpha_{em}(M_Z)}} , \tag{29}$$

where δ represents the experimental uncertainties appearing in (21). In addition δ may also contain SUSY thresholds.

For concreteness and as an illustration let us take the $\mathbb{Z}_8$ orbifold example of [55] with $\mathcal{A}' = -2, \mathcal{B}' = -6$ and $b_1' + b_2' = -10$. It is one of the few orbifolds left over after imposing the conditions on target–space duality anomaly cancellation [55]. To estimate the size of $\triangle$ one may take in eq. (25) $g_{\text{string}} \sim 0.7$ corresponding to $\alpha_{\text{string}} \sim \frac{1}{26}$, i.e. $M_{\text{string}}/M_{\text{GUT}} \sim 20$. Of course this is a rough estimate since M_{string} is determined by the first eq. of (26) together with (25). Nevertheless, the qualitative picture does not change. Therefore to predict the correct low–energy parameter (27) eq. (28) tells us that one needs threshold correction of considerable size:

$$-17.1 \le \triangle \le -16.3 \ . \tag{30}$$

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6 String thresholds

The construction of a realistic unified string model boils down to the question of how to achieve thresholds of that size. To settle the question we need explicit calculations within the given candidate string model. There we can encounter various types of threshold effects. Some depend continuously, others discretely on the values of the moduli fields. For historic reasons we also have to distinguish between thresholds that do or do not depend on Wilson lines. The reason is the fact that the calculations in the latter models are considerably simpler and for some time were the only available results. They were then used to estimate the thresholds in models with gauge group $SU(3) \times SU(2) \times U(1)$ and three families, although as a string model no such orbifold can be constructed without Wilson lines. Therefore, the really relevant thresholds are, of course, the ones found in the (0,2) orbifold models with Wilson lines [47] which may both break the gauge group and reduce its rank. We will discuss the various contributions within the framework of our illustrative model. However the discussion can easily applied for all other orbifolds. The threshold corrections depend on the T and U modulus describing the size and shape of the internal torus lattice. In addition they may depend on non–trivial gauge background fields encoded in the Wilson line modulus B.

Moduli dependent threshold corrections $\triangle$ can be of significant size for an appropriate choice of the vevs of the background fields $T, U, B, \dots$ which enter these functions. Of course in the decompactification limit $T \to i\infty$ these corrections become always arbitrarily huge. This is in contrast to fermionic string compactifications or N=1 sectors of heterotic superstring compactifications. There one can argue that *moduli–independent* threshold corrections cannot become huge at all [56]. This is in precise agreement with the results found earlier in [34, 36]. In field theory threshold corrections can be estimated with the formula [54]

$$\triangle = \sum_{n,m,k} \ln \left(\frac{M^2_{n,m,k}}{M^2_{\text{string}}} \right) \ , \tag{31}$$

with n, m being the winding and momentum, respectively and k the gauge quantum number of all particles running in the loop. The string mass in the $N = 2$ sector of the $\mathbb{Z}_8$ model we consider later with a non–trivial gauge background in the internal directions is determined by [48] :

$$
\begin{aligned}
\alpha' M^2_{n,m,k} &= 4|p_R|^2 \\
p_R &= \frac{1}{\sqrt{Y}} \left[(\frac{T}{2\alpha'}U - B^2)n_2 + \frac{T}{2\alpha'}n_1 - Um_1 + m_2 + Bk_2 \right] \\
Y &= -\frac{1}{2\alpha'}(T - \overline{T})(U - \overline{U}) + (B - \overline{B})^2 \ .
\end{aligned}
\tag{32}
$$

In addition a physical state $|n, m, k, l\rangle$ has to obey the modular invariance condition

$m_1n_1+m_2n_2+k_1^2-k_1k_2+k_2^2-k_2k_3-k_2k_4+k_3^2+k_4^2 = 1-N_L-\frac{1}{2}l_{E_8'}^2$. Therefore the sum in (31) should be restricted to these states. This also guarantees its convergence after a proper regularization. In (31) cancellations between the contributions of various string states may arise. E.g. at the critical point $T=i=U$ where all masses appear in integers of M_{string} such cancellations occur. They are the reason for the smallness of the corrections calculated in [34, 38] and in all the fermionic models [36]. Let us investigate this in more detail. The simplest case $(B=0)$ for moduli dependent threshold corrections to the gauge couplings was derived in [42] :

$$\triangle(T,U) = \ln\left[\frac{-iT+i\overline{T}}{2\alpha'}\left|\eta\left(\frac{T}{2\alpha'}\right)\right|^4\right] + \ln\left[(-iU+i\overline{U})\,|\eta(U)|^4\right] \ . \qquad (33)$$

Formula (33) can be used for any toroidal orbifold compactifications, where the two–dimensional subplane of the internal lattice which is responsible for the N=2 structure factorize from the remaining part of the lattice. If the latter condition does not hold, (33) is generalized [46].

	$T/2\alpha'$	U	$M^2\alpha'$	$ln(M^2\alpha')$	Δ^{II}
Ia	i	i	1	0	-0.72
Ib	$1.25i$	i	$\frac{4}{5}$	-0.22	-0.76
Ic	$4.5i$	$4.5i$	$\frac{4}{81}$	-3.01	-5.03
Id	$18.7i$	i	$\frac{10}{187}$	-2.93	-16.3

Table 1: Lowest mass M^2 of particles charged
under G_A and threshold corrections $\triangle(T,U)$.

In Table 1 we determine the mass of the lowest massive string state being charged under the considered unbroken gauge group G_A and the threshold corrections $\triangle(T,U)$ for some values of T and U.

The influence of moduli dependent threshold corrections to low–energy physics [entailed in eqs. (27)] has until now only been discussed for orbifold compactifications without Wilson lines by using (33). In these cases the corrections only depend on the two moduli T,U. However to obtain corrections of the size $\triangle \sim -16.3$ one would need the vevs $\frac{T}{2\alpha'} = 18.7, U = i$ which are far away from the self–dual points [57, 55]. It remains an open question whether and how such big vevs of T can be obtained in a natural way in string theory.

A generalization of eq. (33) appears when turning on non–vanishing gauge background fields $B \neq 0$. According to (32) the mass of the heavy string states now

becomes B–dependent and therefore also the threshold corrections change. This kind of corrections were recently determined in [47]. The general expression there is

$$\triangle^{II}(T, U, B) = \frac{1}{12} \ln \left[\frac{Y^{12}}{1728^4} |\mathcal{C}_{12}(\Omega)|^2 \right] \ ,$$ (34)

where B is the Wilson line modulus, $\Omega = \begin{pmatrix} \frac{T}{2\alpha'} & B \\ -B & U \end{pmatrix}$ and $\mathcal{C}_{12}$ is a combination of $g = 2$ elliptic theta functions explained in detail in [48]. It applies to gauge groups G_A which are not affected by the Wilson line mechanism. The case where the gauge group is broken by the Wilson line will be discussed later (those threshold corrections will be singular in the limit of vanishing B). Whereas the effect of quantized Wilson lines B on threshold corrections has already been discussed in [38] the function $\Delta^{II}(T, U, B)$ now allows us to study the effect of a continuous variation in B.
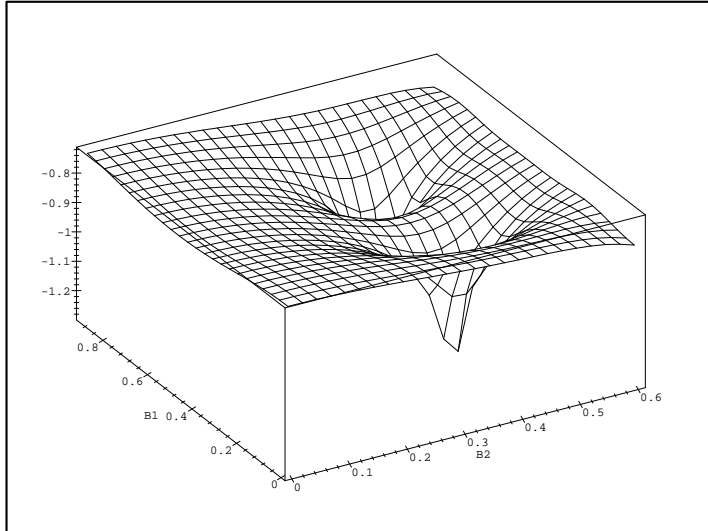

Fig.2 – Dependence of the threshold corrections $\triangle^{II}$
on the Wilson line modulus $B = B_1 + iB_2$ for $\frac{T}{2\alpha'} = i = U$.

We see in Fig.2 that the threshold corrections change very little with the Wilson line modulus B. They are comparable with $\triangle = -0.72$ corresponding to the case of $B = 0$. In this case eq. (34) becomes eq. (33) for $\frac{T}{2\alpha'} = i = U$.

So far all these calculations have been done within models where the considered gauge group G_A is not broken by the Wilson line and its matter representations are not projected out. To arrive at SM like gauge groups with the matter content of the MSSM one has to break the considered gauge group with a Wilson line.

From the phenomenological point of view [58], the most promising class of string vacua is provided by (0,2) compactifications equipped with a non–trivial gauge back-

ground in the internal space which breaks the E_6 gauge group down to a SM–like gauge group [59, 60, 37, 51, 61]. Since the internal space is not simply connected, these gauge fields cannot be gauged away and may break the gauge group. Some of the problems present in (2,2) compactifications with E_6 as a grand unified group like e.g. the doublet–triplet splitting problem, the fine–tuning problem and Yukawa coupling unification may be absent in (0,2) compactifications. It is important that these properties can be studied in the full string theory, not just in the field theoretic limit [59]. The background gauge fields give rise to a new class of massless moduli fields again denoted by B which have quite different low–energy implications than the usual moduli arising from the geometry of the internal manifold itself. In this framework the question of string unification can now be discussed for realistic string models. The threshold corrections for our illustrative model take the form [47]

$$\triangle^I(T, U, B) = \frac{1}{10} \ln \left[Y^{10} \left| \frac{1}{128} \prod_{k=1}^{10} \vartheta_k(\Omega) \right|^4 \right] , \qquad (35)$$

where ϑ_k are the ten even $g = 2$ theta–functions [48]. Equipped with this result we can now investigate the influence of the B–modulus on the thresholds and see how the conclusions of ref. [57, 55] might be modified. The results for a representative set of background vevs is displayed in Fig.3.
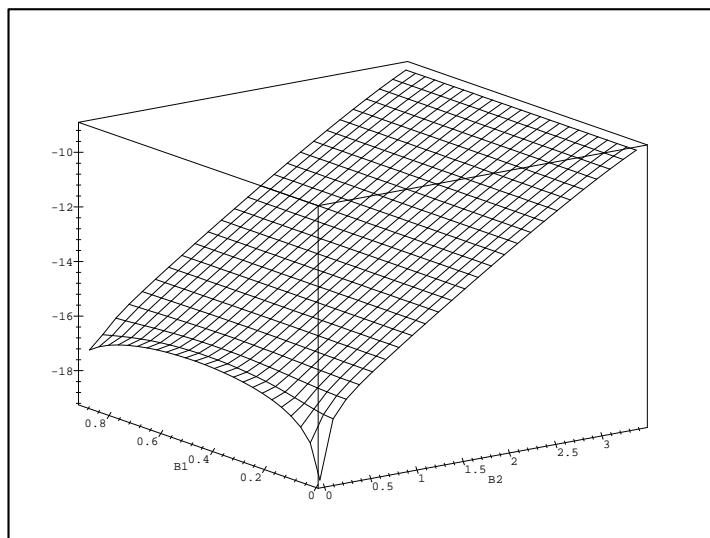

Fig.3 – Dependence of the threshold corrections $\triangle^I$
on the Wilson line modulus $B = B_1 + iB_2$ for $\frac{T}{2\alpha'} = 4.5i = U$.

From this picture we see that threshold corrections of $\triangle \sim -16.3$ can be obtained for the choice of $\frac{T}{2\alpha'} \sim 4.5i \sim U$ and $B = \frac{1}{2}$. This has to be compared to the model

in ref. [55] where such a value was achieved with $T = 18.7i$ and $B = 0$. This turns out to be a general property of the models under consideration. With more moduli, sizeable threshold effects are achieved even with moderate values of the vevs of the background fields.

7 String unification

Equipped with these explicit calculations of string threshold corrections we can now ask the question how string theory might lead to the correct prediction of gauge coupling constants. We also hope to deduce information on the spectrum of theories that lead to successful gauge coupling unification.

The modulus plays the rôle of an adjoint Higgs field which breaks e.g. the $G_A = E_6$ down to a SM like gauge group G_a. According to eq. (32) the vev of this field gives some particles masses between zero and M_{string}. This is known as the stringy Higgs effect. Such additional intermediate fields may be very important to generate high scale thresholds. Sizeable threshold corrections $\triangle$ can only appear if some particles have masses different from the string scale M_{string} and where cancellations between different states as mentioned above do not take place. In particular some gauge bosons of G_A become massive receiving the mass:

$$\alpha' M_I^2 = \frac{4}{Y}|B|^2 \ . \tag{36}$$

As before let us investigate the masses of the lightest massive particles charged under the gauge group G_a. For our concrete model we have $M_{\text{string}} = 3.6 \cdot 10^{17} \text{GeV}$.

	$T/2\alpha'$	U	B	M_I [GeV]	$ln(M_I^2\alpha')$	$\triangle^I$
IIa	i	i	$\frac{1}{10^5}$	$8.4 \cdot 10^{12}$	-23.0	-10.03
IIb	i	i	$\frac{1}{2}$	$4.2 \cdot 10^{17}$	-1.39	-1.72
IIc	$1.25i$	i	$\frac{1}{2}$	$3.7 \cdot 10^{17}$	-1.61	-2.12
IId	$4i$	i	$\frac{1}{2}$	$2.1 \cdot 10^{17}$	-2.78	-7.86
IIe	$4.5i$	$4.5i$	$\frac{1}{2}$	$9.3 \cdot 10^{16}$	-4.39	-16.3
IIf	$18.7i$	i	$\frac{1}{2}$	$1.1 \cdot 10^{16}$	-4.31	-43.3

Table 2: Lowest mass M_I of particles charged
under G_a and threshold corrections $\triangle^I$ for $B \neq 0$.

Whereas Δ^{II} describes threshold corrections w.r.t. to a gauge group which is not broken when turning on a vev of B, now the gauge group is broken for $B \neq 0$ and in particular this means that the threshold $\triangle^I$ shows a logarithmic singularity for $B \to 0$ when the full gauge symmetry is restored. This behaviour is known from field theory and the effects of the heavy string states can be decoupled from the former: Then the part of $\triangle_a^I$ in (23) which is only due to the massive particles becomes [47, 50]

$$\frac{b_A - b_a}{16\pi^2} \ln \frac{M_{\text{string}}^2}{|B|^2} - \frac{b_A'}{16\pi^2} \ln \left| \eta \left(\frac{T}{2\alpha'} \right) \eta(U) \right|^4 , \qquad (37)$$

where the first part accounts for the new particles appearing at the intermediate scale of M_I and the other part takes into account the contributions of the heavy string states. One of the questions of string unification concerns the size of this intermediate scale M_I. In a standard grand unified model one would be tempted to identify M_I with M_{GUT}. While this would also be a possibility for string unification, we have in string theory in addition the possibility to consider $M_I > M_{\text{GUT}}$. The question remains whether the thresholds in that case can be big enough, as we shall discuss in a moment. Let us first discuss the general consequences of our results for the idea of string unification without a grand unified gauge group. Due to the specific form of the threshold corrections in eq. (26) unification always takes place if the condition $\mathcal{A}\mathcal{B}' = \mathcal{A}'\mathcal{B}$ is met within the errors arising from the uncertainties in (21). It guarantees that all three gauge couplings meet at a single point M_X [55]:

$$M_X = M_{\text{string}} \, e^{\frac{1}{2}\frac{\mathcal{A}'}{\mathcal{A}}\triangle} . \qquad (38)$$

For our concrete model this leads to $M_X \sim 2 \cdot 10^{16}$GeV. Given these results we can now study the relation between M_I and M_X, which plays the rôle of the GUT–scale in string unified models. As a concrete example, consider the model IIe in Table 2. It leads to an intermediate scale M_I which is a factor 3.9 smaller than the string scale, thus $\sim 10^{17}$GeV, although the apparent unification scale is as low as 2×10^{16}GeV. We thus have an explicit example of a string model where all the non–MSSM particles are above $9.3 \cdot 10^{16}$GeV, but still a correct prediction of the low energy parameters emerges. *Thus string unification can be achieved without the introduction of a small intermediate scale.*

Of course, there are also other possibilities which lead to the correct low–energy predictions. Instead of large threshold corrections one could consider a non–standard hypercharge normalization, i.e. a $k_1 \neq 5/3$ [62]. This would maintain gauge coupling unification at the string scale *with* the correct values of $\sin^2 \theta_W(M_Z)$ and $\alpha_S(M_Z)$. However, it is very hard to construct such models. A further possibility would be to give up the idea of gauge coupling unification within the MSSM by introducing extra massless particles such as $(\mathbf{3}, \mathbf{2})$ w.r.t. $SU(3) \times SU(2)$ in addition to those of the SM [63, 56]. A careful choice of these matter fields may lead to sizable additional intermediate threshold corrections in (27) thus allowing for the correct low–energy

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data (21). Unfortunately the price for that is exactly an introduction of a new intermediate scale of $M_I \sim 10^{12-14}$GeV. It seems to be hard to explain such a small scale naturally in the framework of string theory. In some sense such a model can be compared to the model IIa in table 2. Other possible corrections to (27) may arise from an extended gauge structure between M_X and M_{string}. However this might even enhance the disagreement with the experiment [56]. Finally a modification of (27) appears from the scheme conversion from the string– or SUSY–based $\overline{DR}$ scheme to the $\overline{MS}$ scheme relevant for the low–energy physics data (21). However these effects are shown to be small [56].

8   Conclusions

We have seen that string unification is easily achieved with moduli dependent threshold corrections within (0,2) superstring compactification. The Wilson line dependence of these functions is comparable to that on the T and U fields thus offering the interesting possibility of large thresholds with background configurations of moderate size. All non–MSSM like states can e.g. be heavier than 1/4 of the string scale, still leading to an apparent unification scale of $M_X = \frac{1}{20}M_{\text{string}}$. We do not need vevs of the moduli fields that are of the order 20 away from the natural scale, neither do we need to introduce particles at a new intermediate scale that is small compared to M_{string}. The situation could be even more improved with a higher number of moduli fields entering the threshold corrections: They may come from other orbifold planes giving rise to N=2 sectors or from additional Wilson lines. We think that the actual moderate vevs of the underlying moduli fields can be fixed by non–perturbative effects as e.g. gaugino condensation.

In this mechanism of supersymmetry breakdown we have seen the central role played by the dilaton. Fixing the vev of the dilaton to a satisfactory value is possible in the presence a nonperturbative modification of the gauge kinetic function that seems to arise in a broad class of string theories. It also leads to a solution of a longstanding problem concerning the overall size of the gauge coupling constant, since it allows weak coupling even for $< S >= 1$.

9   Acknowledgements

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